An Effective Field Theory
at Finite Density

Masayasu Harada[(a)1], Dong-Pil Min[(b)2], Tae-Sun Park[(c)3], Chihiro Sasaki[(a)4] and
Chaejun Song[(d)5]

[(a)] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*
[(b)] *School of Physics, Seoul National University, 151-747, Korea*
[(c)] *School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea*
[(d)] *Department of Physics, Pusan National University, Busan 609-735, Korea*

Abstract

An effective theory to treat the dense nuclear medium by the perturbative expansion method is proposed as a natural extension of the Heavy Baryon Chiral Perturbation Theory (HBChPT). Treating the Fermi momentum scale as a separate scale of the system, we get an improved convergence and the conceptually clear interpretation. We compute the pion decay constant and the pion velocity in the nuclear medium, and find their characters different from what the usual HBChPT predicts. We also obtain the Debye screening

[1]E-mail:harada@eken.phys.nagoya-u.ac.jp

[2]E-mail:dpmin@phya.snu.ac.kr

[3]E-mail:tspark@kias.re.kr

[4]E-mail:sasaki@eken.phys.nagoya-u.ac.jp

[5]E-mail:chaejun@charm.physics.pusan.ac.kr

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scale at the normal nuclear matter density, and the damping scale of the pion wave. Those results indicate that the present theory, albeit its improvement over the HBChPT, has the limitation yet to go over to the medium of about 1.3 times of normal matter density due to the absence of the intrinsic density dependence of the coupling constants. We discuss how we overcome this limitation in terms of the renormalization method.

1   Introduction

Low-energy nuclear processes that involve a few nucleons can be described in a systematic and consistent way, thanks to the heavy-baryon chiral perturbation theory (HBChPT) which has been initiated by the pioneering work of Weinberg [1]. This noble approach has been applied to many low-energy processes with great successes; see, for example, Refs. [2, 3, 4, 5].

Motivated with these successes, many already applied the chiral perturbation theory to dilute systems [6, 7, 8, 9, 10] and also to systems around the saturation density [11, 12]. When the density and the corresponding Fermi momentum p_F increases, however, the convergence of free-space HBChPT becomes questionable. This is because HBChPT is a derivative expansion scheme with the expansion parameter Q/Λ_χ; Q stands for the typical momentum scale (and/or the pion mass) of the process, and $\Lambda_\chi \sim 4\pi f_\pi \simeq 1$ GeV is the chiral scale. In nuclear matter, $Q \sim p_F$ even for small fluctuations whose typical energy-momentum scale is much smaller than p_F. For such fluctuations, the contribution of the nucleons whose momenta far from the Fermi surface is however irrelevant. It is thus desirable to build an effective field theory (EFT) by integrating out such "*massive* degrees of freedom", $|\vec{p}| - p_F > \Lambda$, where Λ is the cutoff of the theory. The resulting EFT has two expansion parameters, Q_{ext}/Λ_χ and p_F/Λ_χ, where Q_{ext} represents the energy-momentum scale of the external probes and/or the pion mass, $Q_{ext} \leq \Lambda$. The resulting EFT is thus meaningful only when $Q_{ext} < \Lambda < p_F$ and $p_F < \Lambda_\chi$. These expansion parame-

ters will emerge naturally in the power counting rule of the matrix elements of the physical processes, and the perturbative expansion will follow this counting rule as is the case in the HBChPT where only the expansion scale is $Q_{ext} < \Lambda_\chi$. As we shall see later, the new expansion scale $p_F < \Lambda_\chi$ could be threatened to approach to one when the density of the nuclear medium becomes large. But as we shall show later, the power counting rule shows that the matrix elements depends on the p_F/Λ_χ to twice of the number of loops that consist only of nucleon lines. This guarantees the rapid convergence in the dilute situation. In this work, however, we limit ourselves to a modest case where $Q_{ext} \ll p_F \ll \Lambda_\chi$, which allows the expansion both in Q_{ext}/Λ_χ and in p_F/Λ_χ. This is the region that can be treated by the HBChPT. However, the EFT that we are developing here has two important advantages over the HBChPT. Firstly, the EFT has only relevant degrees of freedom (integrating out the irrelevant ones), which is the key aspect of EFTs that enables us to describe the nature systematically and consistently in a transparent manner. Secondly, since the convergence with respect to powers of Q_{ext}/Λ_χ is much faster than that of p_F/Λ_χ, substantial reduction of the relevant diagrams can be achieved.

The coefficients of the resulting EFT Lagrangian, *i.e.*, low-energy constants (LECs), will be in general different from those in the vacuum and must be determined from QCD. At this moment, we do not have any suitable ways to match our EFT directly with QCD. Instead we fix them by matching the EFT with the HBChPT; the Fermi momentum of the matching point should be upper-bounded by the validity region of HBChPT and lower-bounded by that of our EFT. Once the LECs are fixed, we determine the p_F-dependences of the physical quantities by including the (nucleon-)loop corrections.

Before going further, we acknowledge that the idea of building EFT near the Fermi surface is not new at all. For example, the familiar Landau-Migdal Fermi theory is one of such. The work of Schwenk *et al.* [13] also is worthy to be noted, where the notion of the V_{low-k} [14] has been applied to derive the parameters of the Landau-Migdal theory. Especially our work shares much in common with the

high density effective theory (HDET) developed by Hong [15] and by Schäfer [16], which is an EFT for quark matter at high-dense region where the matching with the (perturbative) full QCD is possible. Compared to the HDET, our prime interests lie in the intermediate density (rather than the asymptotic) region where the (almost massless) Goldstone bosons exist and the relevant Fermion degrees of freedom are still nucleons. However, our approach may extend its applicability to the region where the chiral quarks are the relevant degrees of freedom. Our present study might be relevant for the recent pionic atom experiment at GSI with heavy nuclei, where the pion decay constant is interpreted to decrease that may signal the partial chiral symmetry restoration in medium [17].

We present in this paper our first step to make the in-medium EFT. Our explicit degrees of freedom are nucleons near the Fermi surface, $|\vec{p}| - p_F \leq \Lambda$ and the Goldstone bosons. As is claimed recently by one of us (MH) [18], the role played by vector mesons at high density should not be underestimated to realize the important phase of matter at the transition, the Vector Manifestation (VM) [19, 20]. Especially, we believe it is very important to understand how this phase is connected from the normal nuclear matter. Our model indeed stems from the purpose of its realization. In the present report, we are however much modest to consider only the pion degrees of freedom. The underlying rationale lies in the decimation process at hand. Our theory will allow the shallow excitation nearby the Fermi surface up to, say, the pionic mass scale. Those modes beyond it will be integrated out from the beginning. In a sense, our model will meet the limit of the applicability if the actual mass of certain integrated-out degrees of freedom becomes lighter as the density increases. This phenomena will emerge in our EFT when the parameters do not satisfy the *naturalness*. In this respect, it will be sought in the forthcoming work to include the chiral partner of pion in the dense limit.

It is worth mentioning some of our ansätze. We are dealing with a dilute system in which loop corrections to the 4-Fermi type interaction are suppressed by some powers of p_F/Λ_χ. Furthermore, the most important channel is assumed to be the

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forward scattering one as in the Fermi liquid [21], which builds Landau's Fermi liquid theory [22]. In the present analysis, we neglect BCS phenomena. The incorporation of the BCS channel is postponed to the next step.

This paper is organized as follows: We give a brief illustration to define the field expressing the fluctuation mode of the nucleon in section 2 and list first few terms in section 3. In section 4, we build the power counting rule for our effective Lagrangian and list more higher order terms. The in-medium modification of the vector correlator is studied shortly in section 5. In section 6, we study the corrections to the pion decay constants and the pion velocity using our EFT. The bare parameters for the decay constants will be determined through the matching between our EFT and the HBChPT. In section 7, we consider the mass(-like) terms and check the change in quark condensate due to mass term in our theory. We summarize our works in section 8.

2   Nucleon and its fluctuation fields

In this section, we will define the field corresponding to soft modes of the fluctuation around the Fermi surface and obtain the higher order correction terms in the present model scheme.

Let us begin with the free Lagrangian for the nucleon without interaction at non-zero density:

$$\mathcal{L} = \bar{\psi} \left(i\partial_\mu \gamma^\mu - m_N + \mu\gamma_0 \right) \psi \ , \tag{2.1}$$

where μ is the baryon chemical potential and m_N the nucleon mass. Fermi momentum $\vec{p}_F$ is related to the mass m_N and the chemical potential μ as in

$$p_F \equiv |\vec{p}_F| = \sqrt{\mu^2 - m_N^2} \ . \tag{2.2}$$

In the dense medium of interacting nucleons, we should use the in-medium nucleon

mass, say m_N^*, instead of the one in vacuum [23]. But for the notational simplicity, we refer to m_N as the in-medium mass m_N^* throughout this paper.

In order to consider the fluctuation mode around the Fermi surface, we introduce the following field ψ_1:

$$\psi(x) = \sum_{\vec{v}_F} e^{i\vec{p}_F \cdot \vec{x}} \psi_1(x; \vec{v}_F) , \tag{2.3}$$

where the Fermi velocity $\vec{v}_F$ is related to the Fermi momentum $\vec{p}_F$ as

$$\vec{v}_F \equiv \frac{1}{\mu} \vec{p}_F . \tag{2.4}$$

Simply the sum of $\vec{v}_f$ means the angle sum on the Fermi surface. By using the field $\psi_1(x; \vec{v}_F)$ in Eq. (2.3), the Lagrangian in Eq. (2.1) is rewritten as

$$\mathcal{L} = \sum_{\vec{v}_F} \bar{\psi}_1 \left(i\partial_\mu \gamma^\mu - \vec{p}_F \cdot \vec{\gamma} - m_N + \mu\gamma_0 \right) \psi_1 , \tag{2.5}$$

where $\vec{\gamma} = (\gamma^1, \gamma^2, \gamma^3)$. To delete off-diagonal components of the γ matrices between $\bar{\psi}_1$ and ψ_1 we introduce the following unitary transformation:

$$\psi_1(x; \vec{v}_F) = U_F(\vec{v}_F) \psi_2(x; \vec{v}_F) , \tag{2.6}$$

where

$$U_F(\vec{v}_F) \equiv \cos\theta - \frac{\vec{v}_F \cdot \vec{\gamma}}{\bar{v}_F} \sin\theta = \cos\theta - \frac{\vec{p}_F \cdot \vec{\gamma}}{p_F} \sin\theta , \tag{2.7}$$

with $\bar{v}_F = |\vec{v}_F|$ and θ satisfying

$$\cos\theta = \sqrt{\frac{\mu + m_N}{2\mu}} , \quad \sin\theta = \sqrt{\frac{\mu - m_N}{2\mu}} . \tag{2.8}$$

By using the field $\psi_2(x; \vec{v}_F)$ in Eq. (2.6), the Lagrangian in Eq. (2.5) is rewritten as

$$\mathcal{L} = \sum_{\vec{v}_F} \bar{\psi}_2 \left[i\partial_0 \gamma^0 + i\partial_j v_F^j + i\partial_j \left(\gamma_\perp^j + \frac{m_N}{\mu} \gamma_{||}^j \right) + \mu\gamma_0 - \mu \right] \psi_2 , \tag{2.9}$$

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where summations over j are implicitly taken and v_F^j denote components of $\vec{v}_F$ as $\vec{v}_F = (v_F^1, v_F^2, v_F^3)$. From now on, the indices i and j run over 1, 2 and 3. Two γ matrices representing the parallel and the orthogonal components to the Fermi velocity, $\gamma_{\parallel}^j$ and $\gamma_{\perp}^j$, are defined by

$$\gamma_{\parallel}^j \equiv \frac{1}{\vec{v}_F^2} v_F^j (\vec{v}_F \cdot \vec{\gamma}) \,, \tag{2.10}$$

$$\gamma_{\perp}^j \equiv \gamma^j - \frac{1}{\vec{v}_F^2} v_F^j (\vec{v}_F \cdot \vec{\gamma}) \,. \tag{2.11}$$

Now, we decompose the ψ_2 field into two parts as

$$\psi_{\pm}(x; \vec{v}_F) \equiv P_{\pm}\, \psi_2(x; \vec{v}_F) \,, \tag{2.12}$$

where the projection operators $P_{\pm}$ are defined as

$$P_{\pm} \equiv \frac{1 \pm \gamma_0}{2} \,. \tag{2.13}$$

ψ_+ and ψ_- represent particle and antiparticle fields in low energy, respectively. By using the fields $\psi_{\pm}(x; \vec{v}_F)$, the Lagrangian in Eq. (2.9) is rewritten as

$$\mathcal{L} = \sum_{\vec{v}_F} \left[\bar{\psi}_+ V_F^\mu\, i\partial_\mu\, \psi_+ + \bar{\psi}_- \left(-\tilde{V}_F^\mu\, i\partial_\mu - 2\mu \right) \psi_- \right.$$
$$\left. + \bar{\psi}_- \left(\gamma_{\perp}^j + \frac{m_N}{\mu}\, \gamma_{\parallel}^j \right) i\partial_j \psi_+ + \bar{\psi}_+ \left(\gamma_{\perp}^j + \frac{m_N}{\mu}\, \gamma_{\parallel}^j \right) i\partial_j \psi_- \right] \,, \tag{2.14}$$

where

$$V_F^\mu = (1, \vec{v}_F) \,, \quad \tilde{V}_F^\mu = (1, -\vec{v}_F) \,. \tag{2.15}$$

The Lagrangian (2.14) shows that the field ψ_+ expresses the fluctuation mode of the nucleon around the Fermi surface, and the field ψ_- corresponds to the anti-nucleon field carrying the effective mass of 2μ which is large compared with the momentum scale we are considering. Then, we can solve out the ψ_- field in studying the low-momentum region to obtain the Lagrangian including the ψ_+ field. It is to be noted that $\partial_0\, \psi_+$ is of same scale as $\partial_i\, \psi_+$, thanks to the projection.

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In the next section, we will construct the general Lagrangian in terms of the ψ_+ field. Here to have an idea on the higher order terms, we solve out the ψ_- field in the Lagrangian (2.14) at tree level. Using equation of motion for the ψ_- field

$$\psi_- = \frac{\gamma_\perp^j + \frac{m_N}{\mu}\gamma_\parallel^j}{i\tilde{V}_F^\mu \partial_\mu + 2\mu} i\partial_j \psi_+ \, , \tag{2.16}$$

we obtain

$$\mathcal{L} = \sum_{\vec{v}_F}\left[\bar{\psi}_+ \, V_F^\mu \, i\partial_\mu \, \psi_+ - \bar{\psi}_+ \frac{(\tilde{\gamma}^j \partial_j)^2}{2\mu} \sum_{n=0}^\infty \left(-\frac{i\tilde{V}_F^\mu \partial_\mu}{2\mu}\right)^n \psi_+\right], \tag{2.17}$$

with $\tilde{\gamma} = \gamma_\perp + \frac{m_N}{\mu}\gamma_\parallel$. This shows that the terms in the right-hand-side (RHS) other than the first term are suppressed by Q/μ, where Q denotes the typical momentum scale we are considering. Therefore, the first term is the leading order term, which leads to the following propagator for the fluctuation field ψ_+ [15]:

$$iS_F(p) = \frac{1}{-V_F \cdot p - i\epsilon p_0} P_+ \, , \tag{2.18}$$

where $\epsilon \to +0$ and P_+ is the projection operator defined in Eq. (2.13). Other terms are regarded as the higher order terms. Thus, the leading correction to the first term of the RHS is already suppressed considerably at any density.

3 Effective Lagrangian

In this section, we will present our effective Lagrangian. As stressed in Introduction, we are considering the moderate density region. So the mesons other than the pion are expected to be still heavier than the pion, i.e., the pion is the only light mesonic degree of freedom realized as the pseudo Nambu-Goldstone (NG) boson associated with the spontaneous chiral symmetry breaking. In the present analysis, therefore, we build an effective Lagrangian including the field expressing the fluctuation of the nucleon around the Fermi surface, ψ_+, together with the NG boson field, based on

the non-linear realization of chiral symmetry by assuming that all the other mesons are integrated out.

As we illustrated in Introduction, we are considering the nucleon carrying the momentum inside the thin shell of size $\Lambda_{||}$ around the Fermi surface. Then, the momentum of the fluctuation parallel to the Fermi velocity at the reference point is restricted to be smaller than $\Lambda_{||}$. Furthermore, we only consider the situation where the momentum of the fluctuation mode perpendicular to the Fermi velocity is also smaller than $\Lambda_{\perp} \sim \Lambda_{||}$ by assuming that effects from mode carrying large perpendicular momenta are already integrated out and included in the parameters of the effective Lagrangian. This idea is based on Landau's Fermi liquid theory where the forward scattering channel is most important[21] and which works well in nuclear physics [22]. And such a scheme is being used in HDET for quark matter [15]. Thus, in the present analysis, we divide the thin shell around the Fermi surface into small boxes, each of which is characterized by the point on the Fermi surface with the characteristic vector, $\vec{v}_F$. Since the size of each box is $\Lambda_{||}\Lambda_{\perp}^2$, the momentum of pion is restricted to be small and the nucleon interacting with pion cannot escape from the box.

Terms without fermion

The pion is introduced as the NG boson associated with the chiral symmetry breaking of $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \to \mathrm{SU(2)_V}$ through the matrix valued variable ξ

$$\xi = e^{i\pi/F_\pi^t} \ , \tag{3.1}$$

where $\pi = \sum_{a=1}^3 \pi_a T_a$ denotes the pion fields, and F_π^t the parameter corresponding to the temporal component of the pion decay constant. Here the wave function renormalization of the pion field is expressed by F_π^t [9, 24]. Under the chiral symmetry, this ξ transforms nonlinearly as

$$\xi \to \xi' = h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}})\, \xi\, g_R = g_{\mathrm{L}}\, \xi\, h^\dagger(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \ , \tag{3.2}$$

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where $h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}})$ is an element of $\mathrm{SU}(2)_{\mathrm{V}}$ to be uniquely determined. There are two 1-forms constructed from ξ:

$$\alpha_A^\mu \equiv \frac{1}{2i} \left(\mathcal{D}^\mu \xi \cdot \xi^\dagger - \mathcal{D}^\mu \xi^\dagger \cdot \xi \right) , \tag{3.3}$$

$$\alpha_V^\mu \equiv \frac{1}{2i} \left(\mathcal{D}^\mu \xi \cdot \xi^\dagger + \mathcal{D}^\mu \xi^\dagger \cdot \xi \right) , \tag{3.4}$$

where $\mathcal{D}_\mu \xi$ and $\mathcal{D}_\mu \xi^\dagger$ are defined as

$$\mathcal{D}_\mu \xi = \partial_\mu \xi + i\xi \mathcal{R}_\mu ,$$
$$\mathcal{D}_\mu \xi^\dagger = \partial_\mu \xi^\dagger + i\xi^\dagger \mathcal{L}_\mu , \tag{3.5}$$

with $\mathcal{L}$ and $\mathcal{R}$ being the external gauge fields for the chiral $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ symmetry. These 1-forms transform as

$$\alpha_A^\mu \to h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \, \alpha_A^\mu \, h^\dagger(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) ,$$
$$\alpha_V^\mu \to h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \, \alpha_V^\mu \, h^\dagger(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) - \frac{1}{i} h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \, \partial^\mu \, h^\dagger(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) . \tag{3.6}$$

By using these 1-forms the general form of the chiral Lagrangian including pion fields can be constructed. The Lagrangian at the leading order of the derivative expansion is given by

$$\mathcal{L}_{A0} = \left[\left(F_\pi^t \right)^2 u_\mu u_\nu + \left(F_\pi^t F_\pi^s \right) \left(g_{\mu\nu} - u_\mu u_\nu \right) \right] \mathrm{tr} \left[\alpha_A^\mu \alpha_A^\nu \right] , \tag{3.7}$$

where $u_\mu = (1, \vec{0})$ indicates the rest frame of medium, and F_π^t and F_π^s denote the parameters corresponding to the temporal and spatial pion decay constants. In our notation $\langle 0 | \partial_\mu A^\mu | \pi \rangle \propto F_\pi^t p_0^2 - F_\pi^s \vec{p}^2 = 0$ for on-shell pions by axial vector current conservation [25] so the pion velocity is expressed as $V_\pi^2 = F_\pi^s / F_\pi^t$. It should be noted that since F_π^t is the wave function renormalization constant of the pion field, the chiral symmetry breaking scale is characterized by

$$\Lambda_\chi \sim 4\pi F_\pi^t. \tag{3.8}$$

Fermion kinetic term

Now, let us consider the terms of the effective Lagrangian including the field for expressing the fluctuation mode of the nucleon around the Fermi surface. In the following, for notational simplicity, we use Ψ for expressing the fluctuation mode which was expressed by ψ_+ field in the previous section. Under the chiral symmetry the fluctuation mode Ψ transforms as

$$\Psi \to h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \Psi \ , \tag{3.9}$$

where $h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \in \mathrm{SU}(2)_{\mathrm{V}}$. Then the kinetic term of the Ψ field is expressed as

$$\mathcal{L}_{\mathrm{kin}} = \sum_{\vec{v}_F} \bar{\Psi} \, V_F^\mu \, iD_\mu \Psi \ , \tag{3.10}$$

where the covariant derivative is defined by

$$D_\mu \Psi = (\partial_\mu - i\alpha_{V\mu}) \Psi \ , \tag{3.11}$$

with $\alpha_{V\mu}$ defined in Eq. (3.4).

Interactions of fermions with mesons

We consider the interactions of Ψ and $\bar{\Psi}$ to the π field. The possible forms of the interaction among Ψ, $\bar{\Psi}$ and α_A^μ defined in Eq. (3.3) are given by

$$\mathcal{L}_A = \sum_{\vec{v}_F} \Big[i \, \kappa_{A0} \bar{\Psi}(\vec{v}_F \cdot \vec{\gamma}) \gamma_5 \alpha_A^0 \Psi + i \, \kappa_{A\parallel} \bar{\Psi} \gamma_\parallel^i \gamma_5 \alpha_{Ai} \Psi + i \, \kappa_{A\perp} \bar{\Psi} \gamma_\perp^i \gamma_5 \alpha_{Ai} \Psi \Big] \ , \tag{3.12}$$

where κ_{A0}, $\kappa_{A\parallel}$ and $\kappa_{A\perp}$ are dimensionless real constants, and $\gamma_\parallel^i$ and $\gamma_\perp^i$ are defined in Eqs. (2.11) and (2.10). [#1] Note that $\bar{\Psi}\alpha_A^0\Psi$, $\bar{\Psi}v_F^i\alpha_{Ai}\Psi$ and $\bar{\Psi}\left[v_F^i\gamma_i\,,\,\alpha_A^j\gamma_j\right]\Psi$ are prohibited by the parity invariance.

[#1] It may be useful to compare the interaction terms in Eq. (3.12) with those reduced from a simple interaction among pion and nucleons given by

$$ig_A\bar{\psi}\gamma^\mu\gamma_5\alpha_{A\mu}\psi \ .$$

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Four Fermi interaction

For constructing four-Fermi interactions, the following relations are convenient:

$$\bar{\Psi}\gamma_0\Psi = \bar{\Psi}\Psi \ , \quad \bar{\Psi}\gamma_i\Psi = 0 \ ,$$

$$\bar{\Psi}\gamma_5\Psi = 0 \ , \quad \bar{\Psi}\gamma_5\gamma_0\Psi = 0 \ ,$$

$$\bar{\Psi}\left[\gamma_0,\,\gamma_i\right]\Psi = 0 \ , \quad \bar{\Psi}\left[\gamma_i,\,\gamma_j\right]\Psi = 2i\varepsilon_{ijk}\,\bar{\Psi}\gamma_5\gamma^k\Psi \ ,$$

$$\bar{\Psi}\left[\gamma_i,\,\gamma_j\right]\gamma_5\Psi = 0 \ , \quad \bar{\Psi}\left[\gamma_0,\,\gamma_i\right]\gamma_5\Psi = 2\bar{\Psi}\gamma_i\gamma_5\Psi \ . \tag{3.13}$$

Although the excitation momentum of the nucleon included in each pair of particle and hole is restricted to be small in the present analysis, Fermi velocity of one pair can be different from that of another pair. Thus, using the relation in Eq. (3.13), we can write the four-fermion interactions as

$$\mathcal{L}_{4F} = \frac{F_S}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\Psi\right)^2 + \frac{F_A}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\gamma_\mu\gamma_5\Psi\right)^2$$

$$+ \frac{F_T}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}V_F^\mu\gamma_\mu\gamma_5\Psi\right)^2 + \frac{G_S}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\vec{\tau}\Psi\right)^2$$

$$+ \frac{G_A}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\gamma_\mu\gamma_5\vec{\tau}\Psi\right)^2 + \frac{G_T}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}V_F^\mu\gamma_\mu\gamma_5\vec{\tau}\Psi\right)^2 \ . \tag{3.14}$$

where $\vec{\tau}/2$ is the generator of SU(2). It should be noticed that the above four-Fermi interactions allow two pairs of particle and hole, each of which resides in different

By performing the same procedure as in section 2, the above term is reduced to

$$ig_A\bar{\Psi}\left[(\vec{v}_F\cdot\vec{\gamma})\gamma_5\alpha_{A0} + \left(\frac{m_N}{\mu}\gamma_\perp^i + \gamma_\parallel^i\right)\gamma_5\alpha_{Ai}\right]\Psi \ ,$$

where $\gamma_\perp^j$ and $\gamma_\parallel^j$ are defined in Eqs. (2.11) and (2.10). Then, when we match the interaction terms in Eq. (3.12) with the above terms, we obtain the following matching conditions:

$$\kappa_{A0} = \kappa_{A\parallel} = g_A \ , \quad \kappa_{A\perp} = \frac{m_N}{\mu}g_A \ .$$

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box. Here we introduce the dimensionless parameters by rescaling the dimensionful parameters by the temporal component of the bare pion decay constant. This is natural since we are constructing the EFT in the chiral broken phase [26].

4 Power counting

In this section, we first present the power counting theorem in our EFT, which provides that the terms listed in the previous section are actually the ones of leading order. Then, we provide the next order terms according to the power counting.

4.1 Power counting

Let us consider the matrix element M with E_N external nuclear lines, E_π external π lines and E_E lines corresponding to the external fields such as $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ in Eq. (3.5). Here, for simplicity we assume that all the mesonic external fields carry the dimension one and that all the nucleonic external fields the dimension $3/2$. [#2]

To derive a power counting rule for M, we first observe that the pion and nucleon propagators that carry the momenta of order of Q are of order of Q^{-2} and Q^{-1}, respectively. Furthermore, we can count loops that consist at least one pion propagator as of order of $\int d^4q \sim \pi^2 Q^4$, where the factor π^2 is due to the angle integration. In fact, these observations can also be applied to free-space HBChPT. In our EFT, the integral is realized as

$$\int d^4p = \sum_{\vec{v}_F} \int d\vec{l}_\perp^2 \int dl_\parallel^2 \sim 4\pi p_F^2 \int d^2l_\parallel \ , \tag{4.1}$$

since the summation over $\vec{v}_F$ and the integral over the perpendicular space corresponds to the area of the Fermi surface. For the integrations over the momentum parallel to the Fermi momentum $l_\parallel$ and the energy l_0, we assign $\int dl_0 \, dl_\parallel \sim \pi Q^2$,

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[#2] We perform the Fourier transformation for each operator associated with the external field.

where the overall factor π can be understood either by the angle integration in 2-D space or by the residue calculation for the integration over l_0.

Recalling the fact that each loop is accompanied by $(2\pi)^{-4}$, we are thus led to the following equation for the counting of a Feynman diagram expressing M,

$$M \propto Q^{-2I_\pi - I_N} \cdot \left(\frac{Q^4}{16\pi^2}\right)^{L-L_N} \left(\frac{Q^2 p_F^2}{4\pi^2}\right)^{L_N} Q^d , \qquad (4.2)$$

where I_π (I_N) denotes the number of the pion (nucleon) propagators, L the number of loops, and d is the sum of the number of derivatives at i-th vertex d_i. L_N is the number of fermionic loops, each of which carries independent Fermi velocity $\vec{v}_F$, i.e., number of summations over $\vec{v}_F$. We note that we are working in the chiral limit, so that the pion mass does not appear in the above formula. Using the techniques developed in HBChPT, we can simplify the above equation as

$$M \sim \left(\frac{Q}{\Lambda_\chi}\right)^\nu \left(\frac{2p_F}{\Lambda_\chi}\right)^{2L_N} , \qquad (4.3)$$

with [#3]

$$\nu = 2 - \left(\frac{E_N}{2} + E_E\right) + 2(L - L_N) + \sum_i \nu_i ,$$
$$\nu_i \equiv d_i + \frac{n_i}{2} + e_i - 2 . \qquad (4.4)$$

Here n_i (e_i) is the number of nucleons (external fields) at i-th vertex. For the detailed derivation of the above counting rule, we refer Ref. [1, 27, 5]. Note that we have replaced $4\pi F_\pi^t$ by Λ_χ.

It is worth while mentioning on the comparison of our counting rule with that of the HDET [16]. HDET has only one scale $p_F = \mu$ and the quark loop contributions related to four-quark interaction is of $\mathcal{O}(1)$ so all such loop contributions should be

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[#3]There is ambiguity for the overall factor, and thus one may replace "$2p_F/\Lambda_\chi$" by p_F/Λ_χ in Eq. (4.3).

summed. However, the same fermionic loop contributions are of $\mathcal{O}((2p_F/\Lambda_\chi)^{2L_N})$ in our theory. Though $2p_F/\Lambda_\chi$ is not related to Q directly, it is a small quantity at moderate densities which satisfy $p_F \ll \Lambda_\chi$. So L_N is also a good counting parameter and one-fermionic loop calculation is enough for moderate density regions, in which

$$Q \ll p_F \ll \Lambda_\chi \tag{4.5}$$

is satisfied. As density increases and p_F becomes larger, we should sum the all fermion loop with the same ν as we should do in HDET. Furthermore, we may have to include other degrees of freedom which become light in the high density region.

4.2 Higher order terms

One can easily check that all terms listed in the previous section are the leading order interactions with $\nu_i = 0$. Here we list higher order terms with $\nu_i = 1$ for the next order calculations.

Terms with two derivatives

Possible terms with two derivatives are given by [4]

$$\frac{C_1}{4\pi F_\pi^t} \bar{\Psi} D^\mu D_\mu \Psi + \frac{C_2}{4\pi F_\pi^t} \bar{\Psi} \left(V_F^\mu D_\mu\right)^2 \Psi - \frac{C_3}{4\pi F_\pi^t} \bar{\Psi} D^0 D^0 \Psi , \tag{4.6}$$

where the covariant derivative acting on Ψ is defined in Eq. (3.11). [5] Note that the term $\bar{\Psi}\left[\gamma^i,\,\gamma^j\right]\left[D_i,\,D_j\right]\Psi$ exists. But this is rewritten as

$$\bar{\Psi}\left[\gamma^i,\,\gamma^j\right]\left[D_i,\,D_j\right]\Psi = -i\,\bar{\Psi}\left[\gamma^i,\,\gamma^j\right]F_{ij}(\alpha_V)\Psi , \tag{4.7}$$

where

$$F_{\mu\nu}(\alpha_V) \equiv \partial_\mu \alpha_{V\nu} - \partial_\nu \alpha_{V\mu} - i\left[\alpha_{V\mu},\,\alpha_{V\nu}\right] \tag{4.8}$$

is the field strength defined from the 1-form $\alpha_{V\mu}$. We will list this term later.

---

[4]Note that the term $\bar{\Psi}\gamma^i D_i \gamma_5 D^0 \Psi$ is prohibited by the parity invariance.

[5]It should be noted that the free fermion model provides $C_1 = C_2 = C_3 = 4\pi F_\pi^t/\mu \sim 1$.

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Terms with one meson field and one derivative

Possible terms including one derivative and one α_A field are given by

$$\frac{h_A}{4\pi F_\pi^t}\bar{\Psi}\left(V_F^\mu D_\mu \alpha_{A\nu}\right)\gamma^\nu \gamma_5 \Psi + \frac{\bar{h}_A}{4\pi F_\pi^t}\bar{\Psi}\left(\tilde{V}_F^\mu D_\mu \alpha_{A\nu}\right)\gamma^\nu \gamma_5 \Psi \ , \tag{4.9}$$

where the derivative D_μ acts on only α_A field as

$$D_\mu \alpha_{A\nu} \equiv \partial_\mu \alpha_{A\nu} - i\left[\alpha_{V\mu},\ \alpha_{A\nu}\right] \ . \tag{4.10}$$

Furthermore, as we showed in Eq. (4.7), we have

$$i\frac{h_{V0}}{4\pi F_\pi^t}\bar{\Psi}\left[\gamma^i,\ \gamma^j\right]F_{ij}(\alpha_V)\Psi = i\frac{h_{V0}}{4\pi F_\pi^t}\bar{\Psi}\left[\gamma^\mu,\ \gamma^\nu\right]F_{\mu\nu}(\alpha_V)\Psi \ . \tag{4.11}$$

Note that there are identities:

$$D_\mu \alpha_{A\nu} - D_\nu \alpha_{A\mu} = \widehat{\mathcal{A}}_{\mu\nu}\ , \tag{4.12}$$

$$F_{\mu\nu}(\alpha_V) = i\left[\alpha_{A\mu},\ \alpha_{A\nu}\right] + \widehat{\mathcal{V}}_{\mu\nu}\ , \tag{4.13}$$

where $\widehat{\mathcal{A}}_{\mu\nu}$ and $\widehat{\mathcal{V}}_{\mu\nu}$ are defined as

$$\widehat{\mathcal{V}}_{\mu\nu} \equiv \frac{1}{2}\left[\widehat{\mathcal{R}}_{\mu\nu} + \widehat{\mathcal{L}}_{\mu\nu}\right] = \frac{1}{2}\left[\xi \mathcal{R}_{\mu\nu}\xi^\dagger + \xi^\dagger \mathcal{L}_{\mu\nu}\xi\right]\ ,$$

$$\widehat{\mathcal{A}}_{\mu\nu} \equiv \frac{1}{2}\left[\widehat{\mathcal{R}}_{\mu\nu} - \widehat{\mathcal{L}}_{\mu\nu}\right] = \frac{1}{2}\left[\xi \mathcal{R}_{\mu\nu}\xi^\dagger - \xi^\dagger \mathcal{L}_{\mu\nu}\xi\right]\ , \tag{4.14}$$

with $\mathcal{R}_{\mu\nu}$ and $\mathcal{L}_{\mu\nu}$ being the field strengths of the external gauge fields $\mathcal{R}_\mu$ and $\mathcal{L}_\mu$. So the h_{V0}-term in Eq. (4.11) can be expressed by the term

$$i\frac{\tilde{h}_{V0}}{4\pi F_\pi^t}\bar{\Psi}\left[\gamma^\mu,\ \gamma^\nu\right]\widehat{\mathcal{V}}_{\mu\nu}\Psi \tag{4.15}$$

and the last term in Eq. (4.16) below.

Terms with two meson fields

Possible terms with including two of α_A field are given by

$$\frac{h^t_{AA}}{4\pi F^t_\pi} \bar{\Psi} \alpha^0_A \alpha^0_A \Psi + \frac{h^s_{AA}}{4\pi F^t_\pi} \bar{\Psi} \alpha^i_A \alpha_{Ai} \Psi + \frac{h^\parallel_{AA}}{4\pi F^t_\pi} \bar{\Psi} \left(\alpha_{Ai} v^i_F \right)^2 \Psi + i \frac{h^a_{AA}}{4\pi F^t_\pi} \bar{\Psi} \left[\alpha^\mu_A \gamma_\mu , \, \alpha^\nu_A \gamma_\nu \right] \Psi \ . \quad (4.16)$$

Six Fermi interaction

Simplest terms expressing the six Fermi interaction are easily obtained by multiplying each term in Eq. (3.14) by $\left(\sum_{\vec{v}_F} \bar{\Psi}\Psi \right)$ or $\left(\sum_{\vec{v}_F} \bar{\Psi} V^\mu_F \gamma_\mu \gamma_5 \Psi \right)$ like $\frac{F_{6SS}}{4\pi (F^t_\pi)^5} \left(\sum_{\vec{v}_F} \bar{\Psi}\Psi \right)^3$. Other possible terms are expressed as

$$\frac{F_{6I}}{4\pi (F^t_\pi)^5} \, i\varepsilon_{\mu\nu\alpha\beta} \, V^\mu_F \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\nu\gamma_5\Psi \right) \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\alpha\gamma_5\Psi \right) \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\beta\gamma_5\Psi \right)$$

$$+ \frac{G_{6I}}{4\pi (F^t_\pi)^5} \, i\varepsilon_{\mu\nu\alpha\beta} \, V^\mu_F \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\nu\gamma_5\vec{\tau}\Psi \right) \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\alpha\gamma_5\vec{\tau}\Psi \right) \left(\sum_{\vec{v}_F} \bar{\Psi}\gamma^\beta\gamma_5\Psi \right) \ . \quad (4.17)$$

5   Vector current correlator

We begin to study our EFT with vector current correlation function. In the present case, the vector correlator is simply expressed by the two-point function of the vector external field $\mathcal{V}_\mu$ defined from the external fields $\mathcal{R}_\mu$ and $\mathcal{L}_\mu$ introduced in Eq. (3.5) as

$$\mathcal{V}_\mu = \frac{1}{2} \left(\mathcal{R}_\mu + \mathcal{L}_\mu \right) \ . \quad (5.1)$$

Applying our power counting (4.4) to $\mathcal{V}_\mu$-$\mathcal{V}_\nu$ two-point function denoted by $\Pi^{\mu\nu}_\mathcal{V}$ where $E_N = 0$ and $E_E = 2$, we have

$$\nu = 2(L - L_N) + \sum_i \left(d_i + \frac{n_i}{2} + e_i - 2 \right) \ . \quad (5.2)$$

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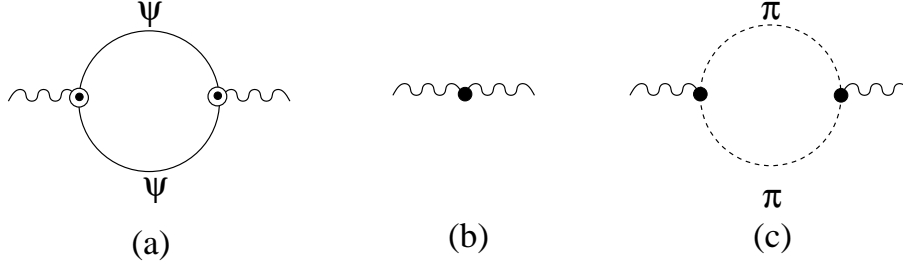
Figure 1: Diagrams contributing to the $\mathcal{V}_\mu$-$\mathcal{V}_\nu$ two-point function: (a) Fermionic one-loop contribution with $\nu = 0$ and $N_L = 1$; (b) Tree contribution with $\nu = 2$ and $N_L = 0$; (c) Pion one-loop contribution with $\nu = 2$ and $N_L = 0$. Here $\odot$ denotes the vertex with V_F^μ and $\bullet$ the momentum dependent vertex.

So the leading loop correction is the fermionic one loop contribution from the covariant kinetic term (3.10), i.e., $\nu = 0, L_N = 1$ (see Fig. 1(a)): [#6]

$$\delta_{ab}\Pi_{\mathcal{V}}^{(1)\mu\nu}(p_0, \vec{p}) = -\mathrm{tr}[T_a T_b] \sum_{\vec{v}_F} 2 V_F^\mu V_F^\nu \int \frac{d^4 l}{i(2\pi)^4} \frac{1}{[-V_F \cdot (l - \eta_1 p) - i\epsilon(l_0 - \eta_1 p_0)]}$$
$$\times \frac{1}{[-V_F \cdot (l + \eta_2 p) - i\epsilon(l_0 + \eta_2 p_0)]} , \tag{5.3}$$

where $u^\mu = (1, \vec{0})$ and η_1 and η_2 are constants satisfying $\eta_1 + \eta_2 = 1$. [#7] In this form the integration over l_0 is finite. Performing the integral over l_0, we obtain

---

[#6]There are no contributions with $\nu = L_N = 0$. The contributions with $\nu = 2$ and $L_N = 0$ are expressed by the sum of the tree diagram and the pion one-loop diagram. The tree diagram given in Fig. 1(b) is constructed from the Lagrangian of the following form:

$$\left[2z_1^L u_\mu u_\alpha g_{\nu\beta} + z_1^T \left(g_{\mu\alpha} g_{\nu\beta} - 2 u_\mu u_\alpha g_{\nu\beta} \right) \right] \mathrm{tr}\left[\hat{\mathcal{V}}^{\mu\nu} \hat{\mathcal{V}}^{\alpha\beta} \right] ,$$

where $\hat{\mathcal{V}}^{\mu\nu}$ is defined in Eq. (4.14). The pion one-loop diagram in Fig. 1(c) is constructed from the leading order Lagrangian in Eq. (3.7).

[#7]In the present integration, we cannot make the shift of the integration momentum in the direction of the Fermi velocity. If we were able to make it, we would be able to show that the integration in Eq. (5.3) vanishes. Then, one might worry that the result of the integral depends on the routing of the momentum in each fermion line, which would imply that the final result depend on η_1 and η_2. However, as we will show below, the dependence on η_1 and η_2 will disappear in the

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$$\Pi_V^{(1)\mu\nu}(p_0, \vec{p}) = -\sum_{\vec{v}_F} V_F^\mu V_F^\nu \frac{1}{V_F \cdot p + i\epsilon p_0}$$

$$\times \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \int \frac{dl_\parallel}{2\pi} \left[\theta\left(-\vec{v}_F \cdot (\vec{l} - \eta_1 \vec{p})\right) - \theta\left(-\vec{v}_F \cdot (\vec{l} + \eta_2 \vec{p})\right) \right] , \quad (5.4)$$

where $\theta(x)$ represents the step function defined by

$$\theta(x) = \begin{cases} 1 & \text{for } x > 0 , \\ 0 & \text{for } x < 0 . \end{cases} \quad (5.5)$$

In Eq. (5.4) we replaced the integration over the spatial momentum as

$$\int \frac{d^3\vec{l}}{(2\pi)^3} = \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \int \frac{dl_\parallel}{2\pi} , \quad (5.6)$$

where

$$l_\parallel \equiv \frac{\vec{v}_F \cdot \vec{l}}{\bar{v}_F} ,$$

$$\vec{l}_\perp \equiv \vec{l} - \vec{v}_F \frac{l_\parallel}{\bar{v}_F} , \quad (5.7)$$

with $\bar{v}_F \equiv |\vec{v}_F|$. The integration over $l_\parallel$ leads to

$$\Pi_V^{(1)\mu\nu}(p_0, \vec{p}) = -\sum_{\vec{v}_F} V_F^\mu V_F^\nu \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \frac{1}{2\pi\bar{v}_F} \frac{\vec{v}_F \cdot \vec{p}}{V_F \cdot p + i\epsilon p_0} . \quad (5.8)$$

We here consider the integration over $\vec{l}_\perp$. Since the momentum $\vec{l}_\perp$ lies on the Fermi surface, the combination of the integration over $\vec{l}_\perp$ and the summation over the Fermi velocity $\vec{v}_F$ implies that the contribution from overall Fermi surface is included. Thus, it is natural to make the following replacement in the above integral:

$$\sum_{\vec{v}_F} \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \Rightarrow \frac{p_F^2}{\pi} \int \frac{d\Omega_{\vec{v}_F}}{4\pi} . \quad (5.9)$$

final result, which indicates that the routing of the loop momentum can be done freely at least at one-loop level.

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Then, we have

$$\Pi_V^{(1)\mu\nu}(p_0,\vec{p}) = -\frac{p_F^2}{2\pi^2 \bar{v}_F} \int \frac{d\Omega_{\vec{v}_F}}{4\pi} V_F^\mu V_F^\nu \frac{\vec{v}_F \cdot \vec{p}}{V_F \cdot p + i\epsilon p_0} \ . \tag{5.10}$$

It should be noticed that the above contribution is covariant under the spatial $O(3)$ rotation after the angle integration is done. Then, we can decompose this as

$$\Pi_V^{(1)\mu\nu}(p_0,\vec{p}) \equiv u^\mu u^\nu \Pi_V^t(p_0,\vec{p}) + (g^{\mu\nu} - u^\mu u^\nu) \Pi_V^s(p_0,\vec{p})$$
$$+ P_L^{\mu\nu} \Pi_V^L(p_0,\vec{p}) + P_T^{\mu\nu} \Pi_V^T(p_0,\vec{p}) \ , \tag{5.11}$$

where $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are the longitudinal and transverse polarization tensors defined by

$$P_L^{\mu\nu} = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - P_T^{\mu\nu} \ ,$$
$$P_T^{\mu\nu} = g_i^\mu \left(\delta_{ij} - \frac{\vec{p}_i \vec{p}_j}{\bar{p}^2}\right) g_j^\nu \ , \tag{5.12}$$

with $\bar{p} = |\vec{p}|$. By performing the angle integration in Eq. (5.10), four components of $\Pi_V^{(1)\mu\nu}$ are expressed as

$$\Pi_V^t(p_0,\bar{p}) = 0 \ ,$$
$$\Pi_V^s(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{6\pi^2} \ ,$$
$$\Pi_V^L(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{2\pi^2} \frac{p^2}{\bar{p}^2} Y(p_0,\bar{p}) \ ,$$
$$\Pi_V^T(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{4\pi^2} + \frac{p_F^2}{4\pi^2 \bar{v}_F} \frac{p_0^2 - \bar{v}_F^2 \bar{p}^2}{\bar{p}^2} Y(p_0,\bar{p}) \ , \tag{5.13}$$

where

$$Y(p_0,\bar{p}) \equiv \int \frac{d\Omega_{\vec{v}_F}}{4\pi} \frac{\vec{v}_F \cdot \vec{p}}{V_F \cdot p + i\epsilon p_0}$$
$$= -1 + \frac{p_0}{2\bar{v}_F \bar{p}} \ln \frac{p_0 + \bar{v}_F \bar{p} + i\epsilon p_0}{p_0 - \bar{v}_F \bar{p} + i\epsilon p_0} \ . \tag{5.14}$$

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When the external vector current is electromagnetic, the vector correlator (5.10) is the photon self-energy and represents the screening effects. Using the formula for the static electric potential energy between two static charges q_1 and q_2,

$$V(r) = q_1 q_2 \int \frac{d^3 p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{r}}}{\vec{p}^2 + \Pi_V^{00}(p_0 \to 0, \vec{p}^2)} \ , \tag{5.15}$$

in Coulomb gauge, the Debye screening mass squared is

$$m_D^2 = e^2 \frac{\mu p_F}{\pi^2} \ . \tag{5.16}$$

which gives the Debye screening mass about $50\,\mathrm{MeV}$ at normal nuclear matter density.

The vector correlator $\Pi_V^{(1)}$ in Eq. (5.11) does not satisfy the current conservation, $p_\mu \Pi_V^{(1)\mu\nu} \neq 0$. Like in HDET [15, 16], this is cured by adding a counter term

$$\mathcal{L}_{CT} = \frac{p_F^2}{4\pi^2 \bar{v}_F} \int \frac{d\Omega_{\vec{v}_F}}{4\pi} \left(\delta_{ij} - \frac{v_F^i v_F^j}{\bar{v}_F^2} \right) \mathrm{tr}\, [\alpha_V^\mu \alpha_V^\mu]$$

$$= \frac{p_F^2}{6\pi^2 \bar{v}_F} (g_{\mu\nu} - u_\mu u_\nu) \mathrm{tr}\, [\alpha_V^\mu \alpha_V^\nu] \ .$$

We note that inclusion of the above counter term does not change the Debey screening mass in Eq. (5.16). As is well known, the Debey mass comes from the longitudinal component Π_V^L, which cannot be expressed by a simple local term in the Lagrangian. The above counter term gives a contribution to the spatial component Π_V^s.

6   Pion decay constants and pion velocity

In this section we study the correction to the pion decay constants and pion velocity. As shown in, e.g., Refs. [18, 20, 24], these physical quantities are extracted from the two-point function of the axial vector external field $\mathcal{A}_\mu$, of which Feynman diagram is identical to the one in Fig. 1(a) with the external axial vector fields instead of $\mathcal{V}_\mu$.

6.1 Fermionic one-loop correction

Now, let us consider the fermionic one-loop correction to the $\mathcal{A}_\mu$-$\mathcal{A}_\nu$ two-point function. The power counting for the axial vector correlator is the same as the vector correlator case (5.2). Then, in the following, we study the fermionic one-loop correction to the $\mathcal{A}_\mu$-$\mathcal{A}_\nu$ two-point function. From the interaction term in Eq. (3.12), the correction with $\nu = 0$ and $N_L = 1$ is expressed as

$$
\begin{aligned}
\delta_{ab}\Pi_A^{(1)\mu\nu}(p_0, \vec{p}) = &-\text{tr}[T_a T_b] \sum_{\vec{v}_F} 2\left[\kappa_{A0}^2 \bar{v}_F^2 u^\mu u^\nu + \kappa_{A\|}^2 g_i^\mu g_j^\nu \frac{v_F^i v_F^j}{\bar{v}_F^2}\right.\\
&\left. + \kappa_{A\perp}^2 g_i^\mu g_j^\nu \left(\delta^{ij} - \frac{v_F^i v_F^j}{\bar{v}_F^2}\right) + \kappa_{A0}\kappa_{A\|}\left(u^\mu g_j^\nu v_F^j + u^\nu g_j^\mu v_F^j\right)\right]\\
&\times \int \frac{d^4 l}{i(2\pi)^4} \frac{1}{[-V_F \cdot (l - \eta_1 p) - i\epsilon(l_0 - \eta_1 p_0)]}\\
&\times \frac{1}{[-V_F \cdot (l + \eta_2 p) - i\epsilon(l_0 + \eta_2 p_0)]}\\
= &-\delta_{ab}\frac{p_F^2}{2\pi^2 \bar{v}_F} \int \frac{d\Omega_{\vec{v}_F}}{4\pi}\left[\kappa_{A0}^2 \bar{v}_F^2 u^\mu u^\nu + \kappa_{A\|}^2 g_i^\mu g_j^\nu \frac{v_F^i v_F^j}{\bar{v}_F^2}\right.\\
&\left. + \kappa_{A\perp}^2 g_i^\mu g_j^\nu \left(\delta^{ij} - \frac{v_F^i v_F^j}{\bar{v}_F^2}\right) + \kappa_{A0}\kappa_{A\|}\left(u^\mu g_j^\nu v_F^j + u^\nu g_j^\mu v_F^j\right)\right]\\
&\times \frac{\vec{v}_F \cdot \vec{p}}{V_F \cdot p + i\epsilon p_0} .
\end{aligned}
\tag{6.1}
$$

The contribution to $\mathcal{A}_\mu$-$\mathcal{A}_\nu$ two-point function at leading order with $\nu = N_L = 0$ is expressed as

$$
\Pi_A^{(0)\mu\nu}(p_0, \vec{p}) = \left(F_\pi^t\right)^2 u^\mu u^\nu + F_\pi^t F_\pi^s \left(g^{\mu\nu} - u^\mu u^\nu\right) .
\tag{6.2}
$$

As is done for the vector current correlator in Eq. (5.11), it is convenient to decompose the sum of the above contributions as

$$
\begin{aligned}
\Pi_A^{\mu\nu}(p_0, \vec{p}) \equiv &\Pi_A^{(0)\mu\nu}(p_0, \vec{p}) + \Pi_A^{(1)\mu\nu}(p_0, \vec{p})\\
= &u^\mu u^\nu \Pi_A^t(p_0, \vec{p}) + (g^{\mu\nu} - u^\mu u^\nu)\Pi_A^s(p_0, \vec{p}) + P_L^{\mu\nu}\Pi_A^L(p_0, \vec{p}) + P_T^{\mu\nu}\Pi_A^T(p_0, \vec{p}) .
\end{aligned}
\tag{6.3}
$$

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From Eqs. (6.1) and (6.2), two components Π_A^t and Π_A^s are obtained as

$$\Pi_A^t(p_0, \vec{p}) = (F_\pi^t)^2 + \frac{p_F^2}{2\pi^2 \bar{v}_F} \left(\kappa_{A0}^2 \bar{v}_F^2 - \kappa_{A0} \kappa_{A\parallel} \right) Y(p_0, \bar{p}) \ , \tag{6.4}$$

$$\Pi_A^s(p_0, \vec{p}) = F_\pi^t F_\pi^s + \frac{p_F^2}{2\pi^2 \bar{v}_F} \left[\frac{1}{3} \left(\kappa_{A\parallel}^2 - \kappa_{A\perp}^2 \right) \right.$$
$$\left. + \left\{ \left(\kappa_{A0} \kappa_{A\parallel} - \frac{\kappa_{A\parallel}^2 - \kappa_{A\perp}^2}{\bar{v}_F^2} \right) \frac{p_0^2}{\bar{p}^2} - \kappa_{A\perp}^2 \right\} Y(p_0, \bar{p}) \ , \tag{6.5}$$

where the function $Y(p_0, \bar{p})$ is defined in Eq. (5.14).

Using the decomposition of $\Pi_A^{\mu\nu}$ in Eq. (6.3), we can construct the axial vector current correlator as

$$G_A^{\mu\nu}(p_0, \vec{p}) = P_L^{\mu\nu} G_A^L(p_0, \vec{p}) + P_T^{\mu\nu} G_A^T(p_0, \vec{p}) \ , \tag{6.6}$$

where

$$G_A^L(p_0, \vec{p}) = \frac{p^2 \Pi_A^t \Pi_A^s}{-[p_0^2 \Pi_A^t - \bar{p}^2 \Pi_A^s]} + \Pi_A^L \ ,$$
$$G_A^T(p_0, \vec{p}) = -\Pi_A^s + \Pi_A^T \ . \tag{6.7}$$

We define the *on-shell* of the pion from the pole of the longitudinal component G_A^L of axial vector current correlator. Since Π_A^t and Π_A^s have imaginary parts, we determine the pion energy E from the on-shell of the real part by solving the following dispersion formula:

$$0 = \left[p_0^2 \operatorname{Re}\Pi_A^t(p_0, \bar{p}) - \bar{p}^2 \operatorname{Re}\Pi_A^s(p_0, \bar{p}) \right]_{p_0 = E} \ . \tag{6.8}$$

The pion velocity is defined by the relation

$$v_\pi(\bar{p}) \equiv E/\bar{p} \tag{6.9}$$

and is then obtained by solving

$$v_\pi^2(\bar{p}) = \frac{\operatorname{Re}\Pi_A^s(p_0 = E, \bar{p})}{\operatorname{Re}\Pi_A^t(p_0 = E, \bar{p})} \ . \tag{6.10}$$

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From the expressions of $\Pi_A^t(p_0, \bar{p})$ and $\Pi_A^s(p_0, \bar{p})$ in Eqs. (6.4) and (6.5), $\mathrm{Re}\Pi_A^t(p_0 = E, \bar{p}) = \mathrm{Re}[f_\pi^t]^2$ and $\mathrm{Re}\Pi_A^s(p_0 = E, \bar{p}) = \mathrm{Re}[f_\pi^t f_\pi^s]$ are expressed as

$$\mathrm{Re}[f_\pi^t]^2 = \mathrm{Re}\Pi_A^t(p_0 = E, \bar{p}) = (F_\pi^t)^2 + \frac{p_F^2}{2V_\pi}\left(\kappa_{A0}^2 \bar{v}_F^2 - \kappa_{A0}\kappa_{A\parallel}\right) X\left(\frac{\bar{v}_F}{V_\pi}\right) , \quad (6.11)$$

$$\mathrm{Re}[f_\pi^t f_\pi^s] = \mathrm{Re}\Pi_A^s(p_0 = E, \bar{p}) = F_\pi^t F_\pi^s + \frac{p_F^2}{6\pi^2 \bar{v}_F}\left(\kappa_{A\parallel}^2 - \kappa_{A\perp}^2\right)$$
$$+ \frac{p_F^2}{2V_\pi}\left\{V_\pi^2\left(\kappa_{A0}\kappa_{A\parallel} - \frac{\kappa_{A\parallel}^2 - \kappa_{A\perp}^2}{\bar{v}_F^2}\right) - \kappa_{A\perp}^2\right\} X\left(\frac{\bar{v}_F}{V_\pi}\right) , \quad (6.12)$$

where $V_\pi \equiv \sqrt{F_\pi^s / F_\pi^t}$ is the bare pion velocity and the function $X(r)$ is defined by

$$X(r) \equiv \frac{1}{\pi^2 r}\left[-1 + \frac{1}{2r}\ln\left|\frac{1+r}{1-r}\right|\right] . \quad (6.13)$$

Note that we have replaced $v_\pi(\bar{p})$ with V_π in the one-loop contribution in Eqs. (6.11) and (6.12) since the difference is of higher order.

The damping factor of the pion, $\gamma(\bar{p})$, can be defined from the imaginary part of the denominator of the longitudinal axial vector current correlator as

$$\gamma(\bar{p}) = \frac{\bar{p}}{2\mathrm{Re}\Pi_A^t(p_0 = E, \bar{p})} \mathrm{Im}\left[\Pi_A^t(p_0 = E, \bar{p}) - \Pi_A^s(p_0 = E, \bar{p})\right] \quad (6.14)$$

By substituting the expressions of $\Pi_A^t(p_0, \bar{p})$ and $\Pi_A^s(p_0, \bar{p})$ in Eqs. (6.4) and (6.5), this is rewritten as

$$\frac{\gamma(\bar{p})}{\bar{p}} = \frac{1}{2\mathrm{Re}\Pi_A^t(p_0 = E, \bar{p})} \frac{V_\pi p_F^2}{2\pi \bar{v}_F^2} \theta(\bar{v}_F - V_\pi)$$
$$\times \left[\left\{\kappa_{A0}^2 \bar{v}_F^2 - \kappa_{A0}\kappa_{A\parallel}\right\} - \left\{V_\pi^2\left(\kappa_{A0}\kappa_{A\parallel} - \frac{\kappa_{A\parallel}^2 - \kappa_{A\perp}^2}{\bar{v}_F^2}\right) - \kappa_{A\perp}^2\right\}\right] . \quad (6.15)$$

6.2   Determination of parameters

We should stress that the parameters of the EFT must be determined from QCD. One way to determine them is the Wilsonian matching [28, 20], in which the current

correlators in the EFT are matched with those obtained in the operator product expansion (OPE) around $1\,\mathrm{GeV}$. As is stressed in Refs. [29, 18], when we make the matching in the presence of hot and/or dense matter, the parameters of the EFT generally depend on the temperature and/or density (the intrinsic temperature and/or density dependence). In the present case, however, the cutoff of our EFT is around a few hundred MeV at most, so that we cannot make the matching between our EFT and QCD directly. So, here we consider the matching between our EFT and the HBChPT to determine the parameters included in our EFT. We require that the temporal and spatial pion decay constants calculated in our EFT agree with those calculated in the HBChPT at a matching point, say ρ_M.

In the HBChPT at one-loop, the temporal and spatial pion decay constants are expressed as [9]

$$f_\pi^{(\mathrm{HB})t} = f_\pi \left[1 + \frac{2\rho}{f_\pi^2} \left(c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \right] \ , \tag{6.16}$$

$$f_\pi^{(\mathrm{HB})s} = f_\pi \left[1 - \frac{2\rho}{f_\pi^2} \left(c_2 - c_3 + \frac{g_A^2}{8m_N} \right) \right] \tag{6.17}$$

where f_π is the pion decay constant at zero density.

The matching conditions are given by equating Eq. (6.11) with Eq. (6.16) and Eq. (6.12) with Eq. (6.17) at $\rho = \rho_M$:

$$\mathrm{Re}[f_\pi^t]^2 \Big|_{\rho=\rho_M} = \left[f_\pi^{(\mathrm{HB})t} \right]^2 \Big|_{\rho=\rho_M} \ ,$$

$$\mathrm{Re}[f_\pi^t f_\pi^s] \Big|_{\rho=\rho_M} = \left[f_\pi^{(\mathrm{HB})t} f_\pi^{(\mathrm{HB})s} \right] \Big|_{\rho=\rho_M} \ . \tag{6.18}$$

The above conditions are not enough to determine all the parameters, so we adopt the following ansatz to determine the values of κ_{A0}, $\kappa_{A\parallel}$ and $\kappa_{A\perp}$ [see footnote #1]:

$$\kappa_{A0} = \kappa_{A\parallel} = g_A \ , \qquad \kappa_{A\perp} = \frac{m_N}{\mu} g_A \ , \tag{6.19}$$

where g_A is the axial coupling. Here we use the following values of m_N and g_A in

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the matter free vacuum: [8]

$$m_N = 939 \, \text{MeV} \,, \qquad g_A = 1.267 \,. \tag{6.20}$$

In the approximation (6.19), the temporal and spatial pion decay constants in Eqs. (6.11) and (6.12) become

$$\text{Re}[f_\pi^t]^2 = (F_\pi^t)^2 - \frac{p_F^2}{2v_{\pi 0}} \frac{m_N^2}{\mu^2} g_A^2 \, X \left(\frac{\bar{v}_F}{v_{\pi 0}} \right) \,, \tag{6.21}$$

$$\text{Re}[f_\pi^t f_\pi^s] = F_\pi^t F_\pi^s + \frac{p_F^2}{6\pi^2} \bar{v}_F g_A^2 - \frac{p_F^2}{2v_{\pi 0}} \frac{m_N^2}{\mu^2} g_A^2 \, X \left(\frac{\bar{v}_F}{v_{\pi 0}} \right) \,. \tag{6.22}$$

It should be noted that the damping factor of pion vanishes when we take the ansatz in Eq. (6.19) with $\mu = \sqrt{p_F^2 + m_N^2}$:

$$\gamma(\bar{p}) = 0 \,. \tag{6.23}$$

Now, by using the matching conditions in Eq. (6.18) together with the ansatz in Eq. (6.19), we determine the values of the parameters F_π^t and F_π^s for given matching density ρ_M. We take the values for the low-energy constants in the HBChPT as $c_2 = 3.2 \pm 0.25 \, \text{GeV}^{-1}$ [30, 31] and $c_3 = -4.70 \pm 1.16 \, \text{GeV}^{-1}$ [32, 33]. Substituting these values into Eqs. (6.16) and (6.17), we can reduce them to [9]

$$f_\pi^{(\text{HB})t} = f_\pi \left[1 - \frac{\rho}{\rho_0} (0.26 \pm 0.04) \right], \tag{6.24}$$

$$f_\pi^{(\text{HB})s} = f_\pi \left[1 - \frac{\rho}{\rho_0} (1.23 \pm 0.07) \right], \tag{6.25}$$

where we take the pion decay constant in the vacuum as $f_\pi = 92.4 \, \text{MeV}$ and the nucleon mass as Eq. (6.20), and ρ_0 denotes the normal nuclear density. When we

[8]In the high density region, m_N should be replaced with m_N^*. Here we consider the density region up until around the normal nuclear matter density, so that the vacuum mass will give a good approximation. The experimental date shows that there is a 10 % decrease in f_π due to the in-medium modification [17]. Thus if m_N^* decreases by 10 %, our results may be changed by a few %.

take the matching density as $\rho_M/\rho_0 = 0.2, 0.3$ and 0.4, the values of F_π^t and F_π^s are obtained by solving the matching conditions Eq. (6.18) as follows:

$$
(F_\pi^t, F_\pi^s) = \begin{cases}
(88.3\,\text{MeV},\ 69.6\,\text{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.2\,, \\
(87.1\,\text{MeV},\ 59.1\,\text{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.3\,, \\
(87.3\,\text{MeV},\ 50.6\,\text{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.4\,.
\end{cases}
\tag{6.26}
$$

6.3 Density Dependence of the Pion Decay Constants and Pion Velocity

In this subsection, we study the pion decay constants and pion velocity in dense matter. Strictly speaking, the parameters of the EFT generally have the intrinsic density dependence. In our construction of the present EFT, we expand the Lagrangian by $1/\mu$ and refer to the coefficient of each power of $1/\mu$ as the parameter. Then, the density dependences of the parameters are moderate. As a result, the density dependences of the physical quantities are dominated by the dense effect from the fluctuating nucleon loop in the density region not so much higher than the matching point $\rho_M \sim (0.2 - 0.4)\rho_0$. Based on this, we use the values of the parameters obtained in the previous subsection to obtain the density dependences of the pion decay constants and pion velocity through the dense loop correction from the fluctuation mode. We show the density dependences of the temporal and spatial pion decay constants in Fig. 2. Until the matching density ρ_M, the behavior of physical quantities is described by the HBChPT. The HBChPT breaks down at the density where f_π^s vanishes, around $0.8\rho_0$. Before the HBChPT breaks down, we should switch the theory from the HBChPT to our EFT in which the dominant density dependences of the physical quantities come from the nucleon fluctuating near the Fermi surface.

As one can see from the expressions of Eqs. (6.16) and (6.17), the density dependence of the pion decay constants in the HBChPT is proportional to p_F^3. This
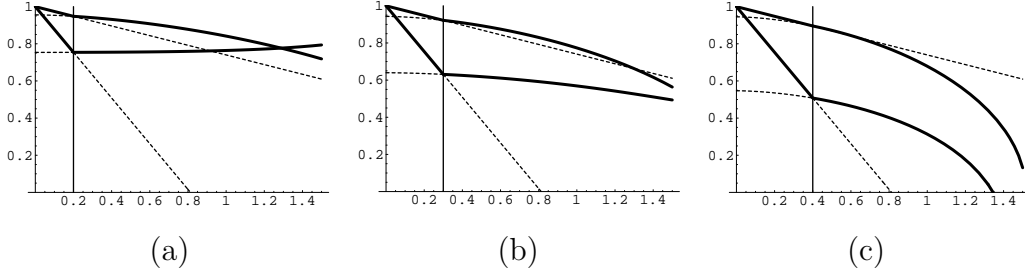
Figure 2: Density dependences of the pion decay constants with the matching point $\rho_M/\rho_0 = 0.2$ (a), 0.3 (b) and 0.4 (c). The horizontal axis shows the value of ρ/ρ_0, and the vertical axis the value of the pion decay constants normalized by the vacuum value. The vertical solid line shows the position of the matching density, ρ_M/ρ_0. In the low-density region, $\rho < \rho_M$, the thick solid lines denote f_π^t/f_π (upper line) and f_π^s/f_π (lower one) obtained from the HBChPT, and the dashed lines denote f_π^t/f_π (upper line) and f_π^s/f_π (lower one) obtained from our EFT. In the higher density region, $\rho > \rho_M$, we used thick solid lines for the predictions obtained from our EFT and dashed lines for those from the HBChPT.

is the reflection that the nucleon inside Fermi sphere contributes to physical quantities. While the pion decay constants in our EFT have the density dependence of p_F^2 [see Eqs. (6.21) and (6.22)], [9] which is a consequence that our EFT includes the effects of the nucleon near Fermi surface. Thus the density dependence of physical quantities becomes gentle and then the available region extends as shown in Fig. 2.

As well as the HBChPT, our EFT also breaks down at the density where f_π^s becomes zero. Figure 2(c) shows that the EFT is not applicable at $\rho \simeq 1.3\rho_0$. Even if f_π^s is still finite, the EFT is not valid at the density where f_π^s becomes larger than f_π^t because of the inconsistency with causality. From Fig. 2(a) and (b), we find that the EFT breaks down at $\rho \simeq 1.3\rho_0$ and $\rho \simeq 1.8\rho_0$, respectively.

Here in order to check the availability of our EFT, we compare one-loop correction with tree contribution. Looking for the density where the ratio of one-loop

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[9] The baryon chemical potential μ and function $X(\bar{v}_F/V_\pi)$ have the implicit p_F dependences. However these density dependences are small as compared with p_F^2.

correction to tree contribution becomes 0.5, we find such density as follows [#10]:
$\rho \simeq 1.2\rho_0$ (for $\rho_M/\rho_0 = 0.3$) and $\rho \simeq 0.8\rho_0$ (for $\rho_M/\rho_0 = 0.4$). As we mentioned
above, Fig. 2(a) shows that our EFT breaks down at $\rho \simeq 1.3\rho_0$. Then in the case
(a) our EFT becomes invalid before perturbative expansion becomes unreliable [The
ratio at $\rho \simeq 1.3\rho_0$ in this case is $0.2 - 0.3$, which is still small]. Thus in any case our
EFT describes physical quantities until around the normal nuclear density ρ_0.

Note that we just calculate $\nu = 0$ and $L_N = 1$. We need to sum all the fermion
loops with the same ν as the density increases. $1.3\rho_0$ indicates how far we can keep
L_N as a counting parameter, not how far we can go with counting parameter ν. By
summing fermion loops with 4-Fermi interactions, we can formally extend our theory
much further. In the high density region, however, the intrinsic density dependence
of the parameters of the EFT will be important. It is crucial for studying the
density dependences of the physical quantities that we include the intrinsic density
dependences of the parameters in some way. Furthermore, we may have to include
other degrees of freedom which becomes light in the high density region.

From Eq. (6.10) and the above results on f_π^t and f_π^s, we can determine the pion
velocity v_π. Figure 3 shows the density dependence of v_π. The pion velocity becomes
smaller with increasing density in low density region. Since the low-energy constant
c_2 in the HBChPT is comparable with tree contribution, the pion velocity largely
decreases. In the region described by our EFT, the pion velocity scarcely change
with respect to density. This behavior is quite similar to the one predicted by the
skyrmion approach in the moderate density region [34].

[#10]The ratio for f_π^t is almost same as that for f_π^s.
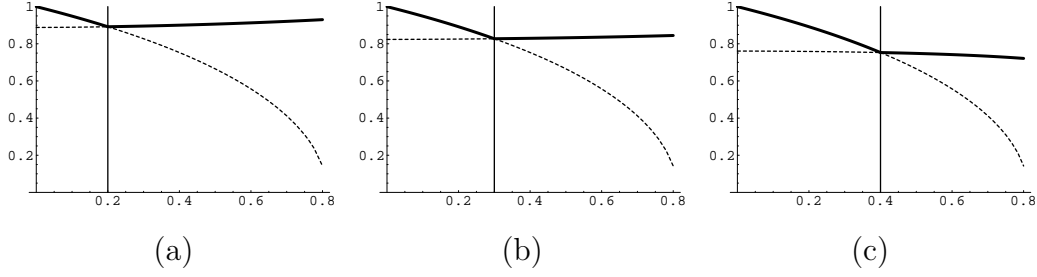
Figure 3: Density dependence of the pion velocity with the matching point $\rho_M/\rho_0 =$ 0.2 (a), 0.3 (b) and 0.4 (c). The horizontal axis shows the value of ρ/ρ_0, and the vertical axis the value of the pion velocity. The position of the matching density, ρ_M/ρ_0, is indicated by the vertical solid lines. For $\rho < \rho_M$, the thick solid line denotes the prediction of the HBChPT and the dashed line the one of our EFT. For $\rho > \rho_M$, on the other hand, we used thick solid line for our EFT and dashed line for the HBChPT.

7 Mass(-like) terms and condensate

7.1 Two mass terms

In section 2, we have deleted the original mass term. But the transformation property of Ψ given in Eq. (3.9) does not prohibit the existence of the term $\bar{\Psi}\Psi$. Then, one might think that we should include the mass term of the fluctuation field Ψ as

$$\mathcal{L}_{\bar{\mu}} = \sum_{\vec{v}_F} \bar{\mu}\bar{\Psi}\Psi \tag{7.1}$$

where $\bar{\mu}$ denotes the mass parameter. Since $\bar{\Psi}\Psi = \Psi^\dagger\Psi$, the effect of $\bar{\mu}$ should be included into the definition of the chemical potential μ and this term should be omitted in the Lagrangian.

Furthermore, we have another type of mass term which comes from the existence of the explicit chiral symmetry breaking due to the current quark mass. In the ordinary chiral perturbation theory the current quark mass is included by the vacuum expectation value of the scalar external source field $\mathcal{S}$. For two flavor case this is

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given by

$$\langle \mathcal{S} \rangle = \mathcal{M} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} . \tag{7.2}$$

In the present analysis we work in the chiral limit, so that we take

$$\langle \mathcal{S} \rangle = 0 . \tag{7.3}$$

This scalar source field $\mathcal{S}$ combined with the pseudoscalar source field $\mathcal{P}$ has the following transformation property under the chiral symmetry:

$$\mathcal{S} + i\mathcal{P} \to g_L \left(\mathcal{S} + i\mathcal{P} \right) g_R^\dagger , \tag{7.4}$$

where g_L and g_R are elements of chiral $\mathrm{SU}(2)_{\mathrm{L,R}}$ groups. To include $\mathcal{S} + i\mathcal{P}$ into the Lagrangian, we define

$$\widehat{\mathcal{M}} \equiv \xi^\dagger \left(\mathcal{S} + i\mathcal{P} \right) \xi^\dagger . \tag{7.5}$$

This $\widehat{\mathcal{M}}$ transforms as

$$\widehat{\mathcal{M}} \to h(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) \cdot \widehat{\mathcal{M}} \cdot h^\dagger(\pi, g_{\mathrm{R}}, g_{\mathrm{L}}) . \tag{7.6}$$

Then, the term including this $\widehat{\mathcal{M}}$ field is written as

$$\mathcal{L}_\mathcal{M} = \sum_{\vec{v}_F} C_\mathcal{M} \, \bar{\Psi} \left[\widehat{\mathcal{M}} + \widehat{\mathcal{M}}^\dagger \right] \Psi , \tag{7.7}$$

where $C_\mathcal{M}$ is a dimensionless real constant. [#11] Note that, for constructing the above term, we used the parity invariance after summing over the Fermi velocity $\vec{v}_F$ (and integrating over $\vec{x}$ as usual): The transformation properties of Ψ and $\widehat{\mathcal{S}}$ under parity are given by

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[#11] We define the chemical potential μ at the chiral limit, so that we include the term in Eq. (7.7) into the Lagrangian.

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$$\Psi(x_0, \vec{x}; \vec{v}_F) \underset{P}{\rightarrow} \Psi(x_0, -\vec{x}; -\vec{v}_F) \,,$$
$$\widehat{\mathcal{M}}(x_0, \vec{x}) \underset{P}{\rightarrow} \widehat{\mathcal{M}}^\dagger(x_0, -\vec{x}) \,. \tag{7.8}$$

Then,

$$\sum_{\vec{v}_F} \int d^4\vec{x}\, \bar{\Psi}(x_0, \vec{x}; \vec{v}_F) \widehat{\mathcal{M}}(x_0, \vec{x}) \Psi(x_0, \vec{x}; \vec{v}_F)$$

$$\underset{P}{\rightarrow} \sum_{\vec{v}_F} \int d^4\vec{x}\, \bar{\Psi}(x_0, -\vec{x}; -\vec{v}_F) \widehat{\mathcal{M}}^\dagger(x_0, -\vec{x}) \Psi(x_0, -\vec{x}; -\vec{v}_F)$$

$$= \sum_{\vec{v}_F} \int d^4\vec{x}\, \bar{\Psi}(x_0, \vec{x}; \vec{v}_F) \widehat{\mathcal{M}}^\dagger(x_0, \vec{x}) \Psi(x_0, \vec{x}; \vec{v}_F) \,. \tag{7.9}$$

Since the current quark masses are assigned as $\mathcal{O}(Q^2)$ in the ordinary chiral perturbation theory, it is natural to assign $\mathcal{O}(Q^2)$ to the field $\widehat{\mathcal{M}}$. Then, we count the term in Eqs. (7.7) as $\mathcal{O}(Q^2)$:

$$\mathcal{L}_\mathcal{M} \sim \mathcal{O}(Q^2) \,, \tag{7.10}$$

or equivalently this term carries $\nu_i = 3$ in the power counting given in Eq. (4.4).

7.2   Quark condensate

The quark condensate can be read from the one-point function of the external source field $\mathcal{S}$. In the present paper we decompose $\mathcal{S}$ as

$$\mathcal{S} = \frac{1}{N_f}\mathcal{S}^0 + \sum_{a=1}^{N_f^2-1} \mathcal{S}^a T_a \,, \tag{7.11}$$

where T_a is the generator of $\mathrm{SU}(N_f)$ normalized as $\mathrm{tr}[T_a T_b] = \frac{1}{2}\delta_{ab}$. Then, the one-point function of $\mathcal{S}^0$ provides the quark condensate per one flavor. From the interaction in Eq. (7.7), one-loop correction to one-point function of $\mathcal{S}^0$ is expressed as

$$\Delta = -4C_{\mathcal{M}} \sum_{\vec{v}_F} \int \frac{d^4 l}{i(2\pi)^4} \frac{1}{-V_F \cdot l - i\epsilon l_0} \ . \tag{7.12}$$

This integral is divergent, so that we here define the regularization to perform the integral. We first replace the momentum integral as in Eq. (5.6). Then, we introduce the cutoff for $l_\parallel$ and l_0 as $-\Lambda_\parallel < l_\parallel < \Lambda_\parallel$ and $-\Lambda_0 < l_0 < \Lambda_0$. By using this regularization method, Eq. (7.12) is written as

$$\Delta = -4C_{\mathcal{M}} \sum_{\vec{v}_F} \int \frac{d^2 \vec{l}_\perp}{(2\pi)^2} \int \frac{dl_\parallel}{2\pi} \theta(\Lambda_\parallel - |l_\parallel|) \int \frac{dl_0}{2\pi i} \theta(\Lambda_0 - |l_0|) \frac{1}{-l_0 + \bar{v}_F l_\parallel - i\epsilon l_0} \ . \tag{7.13}$$

Performing the l_0 integral, we obtain

$$\Delta = -4C_{\mathcal{M}} \sum_{\vec{v}_F} \int \frac{d^2 \vec{l}_\perp}{(2\pi)^2} \int \frac{dl_\parallel}{2\pi} \theta(\Lambda_\parallel - |l_\parallel|)$$
$$\times \left[-\theta(-\bar{v}_F l_\parallel)\theta(\Lambda_0 - |\bar{v}_F l_\parallel|) + \frac{1}{2} \right.$$
$$\left. + \frac{1}{2\pi i} \left\{ \ln\left(\Lambda_0 + (\bar{v}_F l_\parallel)(1 - i\epsilon) \right) - \ln\left(\Lambda_0 - (\bar{v}_F l_\parallel)(1 - i\epsilon) \right) \right\} \right] \ . \tag{7.14}$$

By taking the $\Lambda_0 \to \infty$ limit, this expression is reduced to the following finite form:

$$\Delta = -4C_{\mathcal{M}} \sum_{\vec{v}_F} \int \frac{d^2 \vec{l}_\perp}{(2\pi)^2} \int \frac{dl_\parallel}{2\pi} \theta(\Lambda_\parallel - |l_\parallel|) \left[-\theta(-\bar{v}_F l_\parallel) + \frac{1}{2} \right] \ . \tag{7.15}$$

Now, performing the $l_\parallel$ integral, we obtain

$$\Delta = 0 \ . \tag{7.16}$$

This implies that the number density of the fluctuation mode at one-loop level is zero in chiral limit. It is a kind of a symptom that we choose a proper vacuum in matter.

8 Discussions

As the density of the nuclear medium increases, the Fermi momentum p_F emerges as an additional scale of the system, which deserves to be treated differently from the small scale Q of HBChPT, for a more transparent and better chiral expansion scheme. In this work, we have considered the cases where p_F is much larger than Q but still much smaller than the chiral symmetry breaking scale, $Q \ll p_F \ll \Lambda_\chi$.

We studied the vector and the axial vector correlators in the nuclear medium, and we could calculate the Debye screening mass, pion velocity and the modification of the pion decay constant. The density-dependence of those quantities is found to be highly different from that of the HBChPT, even at the moderate density region. Instead of having the leading linear density dependence as was the case in HBChPT, we have observed rather mild dependence. This may be understood by a following simple consideration: At very low density, the response of the external probe would be proportional to the number of particles in the system, or to the volume of the Fermi sea. When density increases, however, excitations are limited around the Fermi surface due to the Pauli blocking, and the response would now be proportional to the area of the Fermi surface.

The present study enables us to describe the nuclear matter up to the density comparable to ρ_0, the normal nuclear matter density.

To extend the applicable region of the theory, we need to do RPA type of calculations to sum all the contributions with respect to p_F/Λ_χ, for a given order of Q/Λ_χ. This step will result in a very rapid convergence for the processes where $Q/\Lambda_\chi \ll 1$, regardless of the density. The actual calculation can in fact be done rather easily. But this step makes our results to be dependent on the coefficients of the four-fermion interactions. And thus we need a more complete study including the renormalization of those coefficients to have any meaningful predictions. In addition to this, as the density increases, the intrinsic density dependence of our theory can play a crucial role, and thus should be properly treated. The notion of

the *decimation* [35], and the renormalization-group running and additional matching conditions might be necessary to address this issue. Studies on these subjects are left for the future work.

In the high density region in which the RPA is needed, the perturbative structure of the chiral effective theory will be most likely modified with the necessity of choice of the relevant degrees of freedom that we play with. Indeed, we may need to introduce the light vector meson degrees of freedom as claimed to do for the matching with the QCD in the phase of the Vector Manifestation [19]. Even with the chiral partner of the pion is incorporated, the perturbative scheme should stay unchanged as shown in the counting rule.

However, when the chiral phase transition is approached, we have to reconsider the change of the representative scales of the medium from what is considered in this work. Namely, the $4\pi f_\pi$ is reduced eventually to be zero, and thus it cannot serve as the "heavy scale" of the system. In this case, the relevant expansion parameter would be the ratio of the typical momentum transfer compared to the chemical potential, Q/μ, which results in a theory with the same scale dependence as the HDET [16]. The transmutation of the scales could be realized by considering the second decimation as is suggested by Brown *et al* [36].

Acknowledgment

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